Properties of Parallel Lines

Application
Here are two geometric patterns that are used for quilts. Each is a regular hexagon that can be translated and repeated many times to cover a quilt top. Each hexagon contains six shaded trapezoids.

Understanding the Main Ideas

Given: two lines $m$ and $n$ cut by a transversal $t$

- **Corresponding** $\angle 1 \cong \angle 5$
- **Alternate Interior** $\angle 3 \cong \angle 6$
- **Alternate Exterior** $\angle 4 \cong \angle 5$
- **Same-Side Interior** $\angle 1 \cong \angle 5$
- $m \angle 3 + m \angle 5 = 180^\circ$
- $m \angle 4 + m \angle 6 = 180^\circ$

Theorems about parallel lines and transversals

**Alternate Interior Angles Theorem**
If two parallel lines are intersected by a transversal, then alternate interior angles are congruent.

If $k \parallel k$, then $\angle 1 \cong \angle 3$.

**Same-Side Interior Angles Theorem**
If two parallel lines are intersected by a transversal, then same-side interior angles are supplementary.

If $k \parallel k$, then $m \angle 4 + m \angle 5 = 180^\circ$.

Example 1

a. Find the measures of $\angle 1$ and $\angle 2$.

b. Find the values of $x$ and $y$.

- **Solution**

  a. By the Alternate Interior Angles Theorem, $m \angle 1 = 75^\circ$.
     
     By the Same-Side Interior Angles Theorem, $m \angle 2 = 180^\circ - 75^\circ$, or $105^\circ$.

  b. By the Same-Side Interior Angles Theorem, $x^\circ = 180^\circ - 126^\circ$, or $x = 54$.
     
     By the Alternate Interior Angles Theorem, $2y^\circ = 126^\circ$ and therefore $y = 63$. 
Use the diagram at the right. Find the measure of each angle.

1. $\angle 1$

2. $\angle 2$

3. $\angle 3$

4. $\angle 4$

Find the values of $x$, $y$, and $z$.

5. \[ \begin{array}{c}
2x \\
70° \\
x \\
y \\
93° \\
z 
\end{array} \]

6. \[ \begin{array}{c}
140° \\
2x \\
z \\
z \\
(z+10°) 
\end{array} \]

Find each angle measure.

7. $m\angle 1$ ______

8. $m\angle 2$ ______

9. $m\angle ABC$ ______

10. $m\angle DEF$ ______

11. Ocean waves move in parallel lines toward the shore. The figure shows Sandy Beaches windsurfing across several waves. For this exercise, think of Sandy’s wake as a line. \( m\angle 1 = (2x + 2y)^\circ \) and \( m\angle 2 = (2x + y)^\circ \).

\[ x = \]______

\[ y = \]______

12. The endpoints of $\overline{AB}$ are $A(1, 0)$ and $B(7, -1)$. Find the midpoint and length of $\overline{AB}$.
Conditions for Parallel Lines

Application
In order for the oars on each side of this boat to be parallel, corresponding angles formed by the oars and the centerline of the boat must be congruent.

Understanding the Main Ideas
Proving that lines are parallel
You can use the following three converses to prove that two lines are parallel.

Converse of the Corresponding Angles Postulate
If two lines are intersected by a transversal and corresponding angles are congruent, then the lines are parallel.

Converse of the Alternate Interior Angles Theorem
If two lines are intersected by a transversal and alternate interior angles are congruent, then the lines are parallel.

Converse of the Same-Side Interior Angles Theorem
If two lines are intersected by a transversal and same-side interior angles are supplementary, then the lines are parallel.

Example
a. Tell which theorem or postulate you can use to prove that $j \parallel k$.

b. Find the value of $x$ that will allow you to prove that $m \parallel n$.

Solution
a. Converse of the Corresponding Angles Postulate
b. If same-side interior angles are supplementary, then the lines are parallel. So,

\[
6x + (80 - x) = 180
\]

\[
5x + 80 = 180
\]

\[
5x = 100
\]

\[
x = 20
\]
State the theorem or postulate (angle relationship) that proves that \( j \parallel k \).

1. \[ 115^\circ \quad 65^\circ \]
2. \[ \_ \quad \_ \]
3. \[ \_ \quad \_ \]

Tell whether lines \( m \) and \( n \) are parallel or not parallel. Explain your reasoning.

4. \[ \_ \quad \_ \quad 95^\circ \]
5. \[ \_ \quad 102^\circ \quad \_ \]
6. \[ \_ \quad \_ \quad \_ \quad 125^\circ \]

For Exercises 7–9, find the value of \( x \) that will allow you to prove that two lines are parallel.

7. \[ \_ \quad \_ \quad (x + 60)^\circ \quad 50^\circ \]
8. \[ \_ \quad \_ \quad (x + 40)^\circ \quad \_ \quad \_ \quad \_ \quad \_ \]
9. \[ \_ \quad \_ \quad \_ \quad \_ \quad (2x - 10)^\circ \quad (65 - x)^\circ \]

For Exercises 10–15, use the diagram at the right to answer each question. Then tell which postulate or theorem you used.

10. If \( j \parallel k \) and \( \angle 1 = 105^\circ \), what is \( m \angle 13 \)?

11. If \( \angle 9 = \angle 15 \), which lines are parallel?

12. If \( \angle 3 \) and \( \angle 16 \) are supplementary, which lines are parallel?

13. If \( m \parallel n \) and \( m \angle 4 = 75^\circ \), what is \( m \angle 5 \)?

14. If \( j \parallel k \) and \( m \angle 12 = 75^\circ \), what is \( m \angle 6 \)?

A. Corresponding Angles  
B. Alternate Interior Angles  
C. Same-Side Interior Angles

15. If \( m \angle 10 = m \angle 14 \), which lines are parallel?
Use the figure for Exercises 16-23.
Tell whether lines $m$ and $n$ must be parallel from the given information.
If they are, state your reasoning.
(Hint: The angle measures may change for each exercise, and the figure is for reference only.)

16. $\angle 7 \equiv \angle 3$

17. $m\angle 3 = (15x + 22)\degree$, $m\angle 1 = (19x - 10)\degree$, $x = 8$

18. $\angle 7 \equiv \angle 6$

19. $m\angle 2 = (5x + 3)\degree$, $m\angle 3 = (6x - 5)\degree$, $x = 14$

20. $m\angle 8 = (6x - 1)\degree$, $m\angle 4 = (5x + 3)\degree$, $x = 9$

21. $\angle 5 \equiv \angle 7$

22. $\angle 1 \equiv \angle 5$

23. $m\angle 6 = (x + 10)\degree$, $m\angle 2 = (x + 15)\degree$

24. Sketch the boat and oars shown in the Application. Mark the corresponding angles that must be congruent in order for the oars on each side of the boat to be parallel.